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Direct Experimental Evidence for an Atmospheric Gravity Wave Cascade

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13. ABSTRACT *(Maximum 200 words)*

The saturated-cascade theory for atmospheric gravity wave (GW) spectra combines the physics of wave saturation with that of wave-wave interactions (the latter being in the form of a cascade). This theory successfully explained the slopes of 11 classes of measured GW spectra, the variability of some spectra compared to the quasi-universality of certain other spectra, and finally, the observed association between wavelength and wave period in individual wave observations. Unfortunately, the cascade properties in this theory have heretofore received only indirect confirmation in spite of the fact that direct evidence could be obtained by experimental means using today's technology. In this report, the available direct experimental evidence for the wave cascade that is now available in the papers by Nastrom et al. and Bacmeister et al. is examined, and further tests along these lines are suggested.

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1. INTRODUCTION

The saturated cascade (S-C) theory of atmospheric gravity waves (GW's) in Dewan (1994, 1997) utilized both "saturation" and "cascade" concepts to predict the power spectral densities (PSD's) of vertical and horizontal wave numbers, and temporal frequencies of temperature, density, and wind fluctuations due to GW's. "Saturation" explained the quasi-universality of the vertical wave number PSD's of horizontal wind, temperature, and density while "cascade" explained the very much larger variability of all the other PSD's. The combination of both saturation and cascade concepts also resulted in an explanation of the observed wave period-wavelengths seen in the data of Gardner and Voelz (1987), Reid (1986), and Manson (1990). At present, no direct test of the cascade part of the S-C theory has been performed. This would consist of the simultaneous measurement of both the average turbulent dissipation rate, $\bar{\varepsilon}$, and certain of the GW PSD's strongly dependent on $\bar{\varepsilon}$ (see Dewan, 1997). For this reason it is desirable to find other evidence for the cascade in the existing experimental literature. Therefore, in this report the horizontal wavenumber PSD's of wind fluctuations reported in Nastrom et al (1987) and Bacmeister et al (1996) are examined and the discontinuous slopes of these spectra are explained in a manner which is in quantitative agreement (so far as estimation permits without further data) with the S-C model and the cascade property in particular.

The cascade described in Dewan (1997) starts with a "source" at the large scale, low wavenumber denoted by k_{Z*}. The growth of the waves, as they ascend, provides the energy per unit mass at this scale. The cascade goes from this large scale to smaller and smaller scale (and period) waves. As the wave field ascends, k_{Z*} decreases as explained in Smith et al (1987).

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2. EXPERIMENTAL EVIDENCE

Figure 1, from Nastrom et al (1987) shows horizontal wavenumber spectra from horizontal velocity profiles based on the well known GASP program measurements made from a commercial aircraft. These data were selected to ascertain terrain effects and do not reflect an average on all data sets. Our Figure 1 (their Figure 6) was further restricted to the "high wind speed cases" over mountains (see Nastrom et al (1987)) and was further segmented to reflect tropospheric and stratospheric regions. As these authors pointed out there are transition scales where the PSD slopes change from -5/3 to -3 as one progresses to higher wave numbers. These occurred at 23 km in the troposphere and 15 km in the stratosphere. In contrast, Figure 2, taken from Bacmeister et al (1996) made similar measurements from an ER-2 (also called a U-2) aircraft at 20 km altitude. (This is their Figure 14b). They also observed a change from -5/3 to -3 with the steeper slope associated with scales smaller than 3 km. (See their abstract.) The three observations of the scale of the breaks, which occurred at wavelengths of 25 km in the troposphere, 15 km in the stratosphere, and 3 km much higher in the stratosphere suggest that, in accordance with Dewan, 1997, a cascade (showing a -5/3 slope) takes place from higher scales to lower scales as the gravity waves propagate upwards in altitude. The main sources are assumed to be located below the regions of observation. This phenomenon was predicted and noted in Dewan (1997) and is based on the "cascade set up time" hypothesis of W. Hocking, (private communication, 1996).

In other words the spectral break occurs because as the cascade is "set up", that is, extends to sequentially smaller scales, it takes an extended time before it reaches the smallest wave scales. Note that previous theories of the break label the break wavenumber as k_*. It is important to avoid this confusion. Note also that k_* decreases with altitude in contradiction to the observation that the k of the break increases with altitude, thus disproving these previous theories.

It should be mentioned that the majority of experimental horizontal wavenumber spectra of winds do not contain breaks from -5/3 to -3 slopes (see Nastrom and Gage (1985) and U-2 stratospheric observations (20 km) of Crooks et al (1968)). Instead, the breaks seem to occur only when there is an obvious, relatively near, source of gravity waves as in the case of Nastrom et al (1987). In future experiments this can be investigated further. Additionally one would also

expect that the presence of breaks would be seen more often at high altitudes since, for example, doubling the altitude would quadruple the area of exposure to wave sources below.

The specific goal here is to show that the time required for a GW cascade field to ascend from 13 km, i.e. the nominal stratospheric altitude in Nastrom et al (1985) (private communication with Nastrom, 1998), to the higher stratospheric altitude (20 km) of Bacmeister et al (1996) is about equal to the estimated time needed for the cascade to progress from the 15 km lower stratospheric scale to the 3 km upper stratospheric scale. This, as will be shown, therefore quantitatively explains the scale of discontinuous change in PSD slopes and its altitude dependence as shown here in Figures 1 and 2.

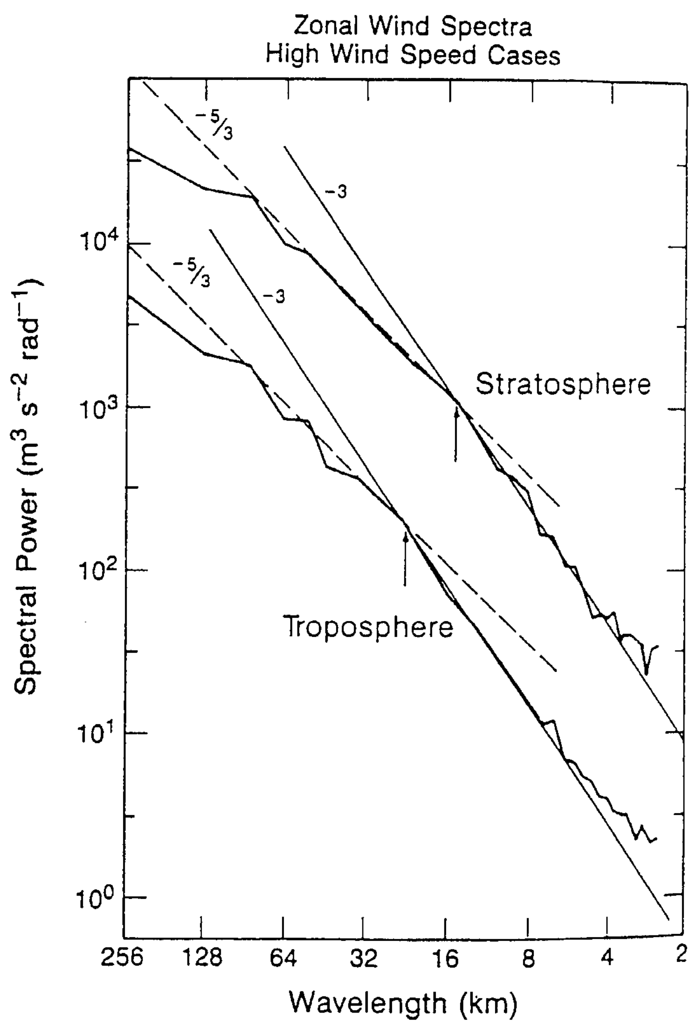

Figure 1. Horizontal wavenumber spectra of zonal winds (high wind speed case over mountains) from tropospheric and stratospheric measurements. Note the scales of the breaks where the slopes change from -5/3 to -3. This figuure is from Nastrom et al (1987) and it is their Figure 6.

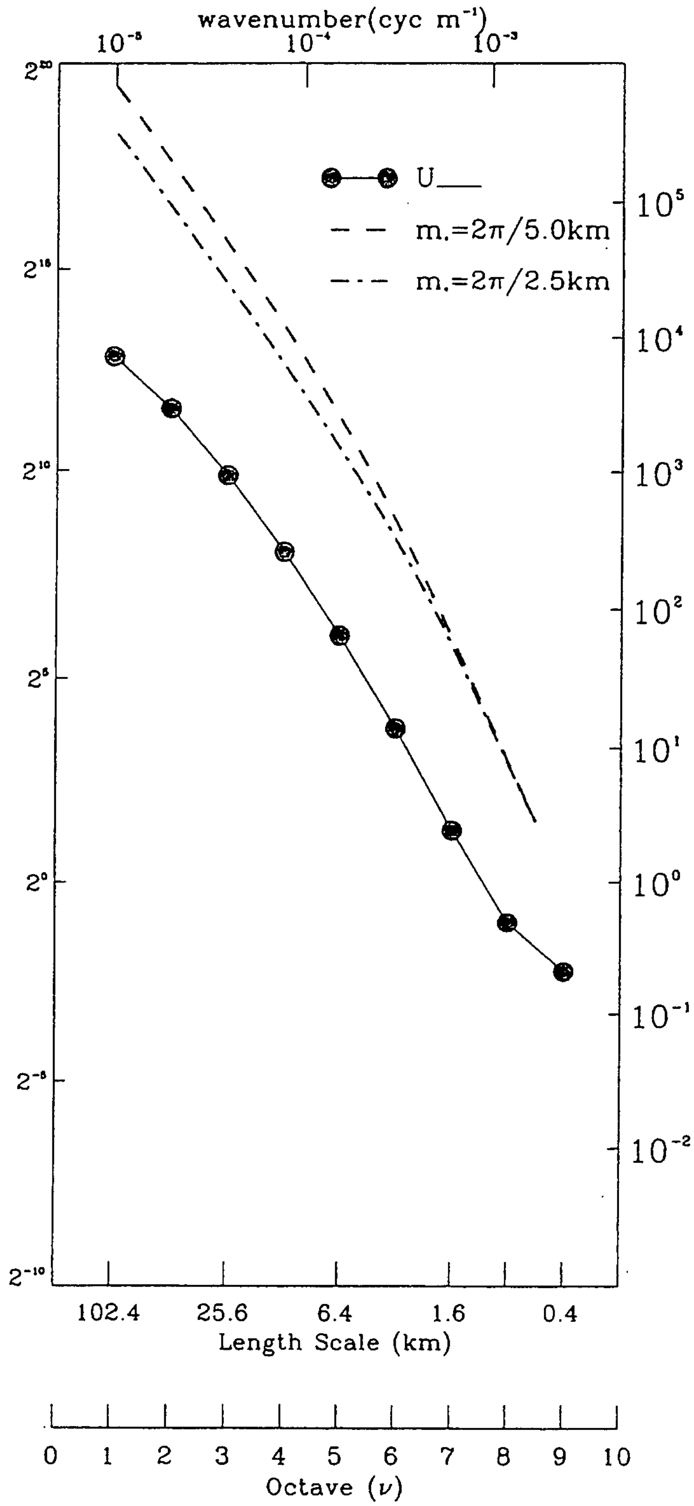

Figure 2. Horizontal wavenumber spectrum of horizontal winds at 20 km altitude from Bacmeister et al (1996). It is their Figure 14b. There is a break from a slope near -5/3 to one significantly larger at around 3 km.

3. QUANTITATIVE TREATMENT USING THE S-C MODEL

The quantitative model to be described requires the knowledge of a number of parameters. Since they are not presently available it will be necessary to use nominal values based on past observations. They could be obtained in future experiments. These are listed in Table 1B (based on Dewan (1997)). In other words, only these already published values, rather than values "tuned" to Figures 1 and 2, are used. Our present purpose is merely to see if the model gives results that are within reason. Table 1A also gives the required equations from Dewan (1997).

Table 1.

A. Equations (from Dewan (1997), Table 1). (See B below for constants).	
Horizontal velocity horizontal wavenumber spectrum	$\psi^{(e)}_{v_x}(k_x) = \frac{\alpha \bullet (\varepsilon\, a_2/a_1)^{2/3}}{3} (2\pi)\, k_x^{-5/3}$
Vertical velocity horizontal wavenumber spectrum	$\psi^{(e)}_{v_z}(k_x) = \frac{\alpha \bullet (\varepsilon\, a_2/a_1)^{4/3}}{3N^2} (2\pi)\, k_x^{-1/3}$
Horizontal velocity frequency spectrum	$\psi_{v_x}(\omega) = \frac{\alpha \bullet (\varepsilon\, a_2/a_1)}{2} (2\pi)\, \omega^{-2}$
Characteristic vertical wavenumber	$k_{z_*} = \left(\frac{fN^2}{(\varepsilon a_2/a_1)}\right)^{1/2}$
Saturated cascade relation	$k_z^3 = \frac{N^3}{(\varepsilon a_2/a_1)}\, k_x$
Wavenumber bandwidth (from Eq. (21), Dewan (1997))	$\Delta k_z = \frac{k_z}{4n}$
Wavelength-period relation	$\tau^{3/2} = \left(\frac{2\pi}{\varepsilon a_2/a_1}\right)^{1/2} \lambda_x$

B. Parametric values (Dewan (1997), Table 2)	
Universal constants	$\begin{cases} \alpha = \frac{1}{6} \\ a_2/a_1 = 2.5 \times 10^3 \\ n = 4 \end{cases}$
Stratospheric dissipation rate (volume and time average)	$\bar{\varepsilon} = 10^{-5}\ m^2/s^3$
Mesospheric dissipation rate (volume and time average)	$\bar{\varepsilon} = 1.5 \times 10^{-4}\ m^2/s^3$
Stratospheric { Brunt frequency Brunt period	$N = 0.02$ rad/s $\tau_B = \frac{2\pi}{N} = 314$ s
43° latitude (representative) inertial frequency	$f = 10^{-4}$ rad/
Characteristic vertical wavenumbers based on Smith et al (1987)	Tropospheric $k_{z_*} = 2\pi/1$km Stratospheric $k_{z_*} = 2\pi/5$ km

The cascading GW field in Dewan (1994, 1997) originates with the largest horizontal wavelengths (λ_x) and periods (τ) and cascades downwards to the small wavelengths and periods. An estimate of the vertical ascent rate of this GW field would be of the order of the vertical group velocity υ_{g_z} of the inertial waves (see Dewan, 1997). This υ_{g_z} is sufficiently well approximated by

$$\upsilon_{g_z} = \frac{\lambda_{z_*}}{\tau_f} \tag{1}$$

where λ_{z_*} is the longest S-C wavelength or the "characteristic" wavelength (c.f. Smith et al (1987) and Dewan (1997)) and τ_f is the inertial period. Let TOF equal the "time of flight" or ascent time for the GW field to go from 13 km to 20 km. Then

$$\mathrm{TOF} = \frac{(20\ \mathrm{km} - 13\ \mathrm{km})}{\upsilon_{g_z}} . \tag{2}$$

At 43° Lat., $\tau_f = \frac{2\pi}{f} = 6.28 \times 10^4$s approximately from Table 1B. The nominal value for λ_{z*} in the stratosphere from Smith et al (1987) and Table 1B is 5 km. Using these in Equation (1) and Equation (2) gives $\upsilon_{g_z} = 8 \times 10^{-2}$ m/s and TOF = 24.3 hrs respectively.

We next compare this to the "cascade duration", CD, needed to propagate down in scale from $\lambda_x = 15$ km (seen at 13 km altitude) to $\lambda_x = 3$ km (seen at 20 km altitude). The following calculations depend crucially on information in Dewan (1997) and the reader must refer to Table 1A for the equations used below. Thus the cascade duration is given by

$$\mathrm{CD} = \sum_{i=0}^{J} n\,\tau_i \tag{3}$$

where τ_i is the period of the i^{th} wave mode, n is the number of periods needed for one mode (e.g. the $(i-1)^{th}$ mode) to decay and to simultaneously "feed" the next smaller mode (e.g. the i^{th} one), and J is the total number of "cascade steps" (in the model) undergone during the cascade duration under investigation (c.f. Dewan (1997)). In other words J is the number of modes involved in going from the initial (larger) scale to the final (smaller) scale as the wave field ascends.

Table 1A contains the wavelength-period relation

$$\tau_i = \left(A\,\lambda_{x_i}\right)^{2/3} \tag{4}$$

where λ_x refers to horizontal wavelength and where

$$A \equiv \sqrt{\frac{a_1\, 2\pi}{a_2\, \bar{\varepsilon}}} \tag{5}$$

where, again, $\bar{\varepsilon}$ is the time and volume average turbulent dissipation rate, and the ratio (a_1/a_2) is a universal constant in the model. (See Table 1.) From Table 1, $a_2/a_1 = 2.5 \times 10^3$. Also, from Table 1 an appropriate nominal average value for stratospheric $\bar{\varepsilon}$ is 10^{-5} m^2/s^3. (It would be highly desirable that average ε be measured simultaneously with the PSD's in question in a future experiment.) These nominal values give one A = 15.9 which should be regarded as a "rough" estimate to be tolerated on the basis that it is the best available for the present. Thus one can estimate τ_i for any given λ_{x_i} from Equation (4).

The next step in our estimation of CD from Equation (3) is to obtain J as well as the values of the intermediate λ_i s (and τ_i s) of each step in the model cascade. For this we start with the mode bandwidths given in Table 1A by

$$\Delta k_z = \frac{k_z}{4n} \tag{6}$$

where k_z is the vertical wave number, $2\pi/\lambda_z$, and where Δk_z is the bandwidth of the mode in question. However, Equations (3) and (4) require λ_{x_i} and $\Delta\lambda_{x_i}$ (i.e. the bandwidth in terms of the horizontal wavelength).

For this purpose we utilize

$$k_z = \left(\frac{a_1 N^3}{a_2 \bar{\varepsilon}}\right)^{1/3} k_x^{1/3} \equiv G^{1/3} k_x^{1/3} \tag{7}$$

where k_x is $2\pi/\lambda_x$ and where Equation (7) is given in Table 1 above. We use Equation (7) in order to relate $\lambda_{x_{i+1}}$ to λ_{x_i} (i.e. the wavelengths of contiguous modes separated by bandwidth $\Delta\lambda_{x_i}$). We start by defining

$$\lambda_{x_{i+1}} \equiv \lambda_{x_i} + \Delta\lambda_{x_i}. \tag{8}$$

In other words, $\Delta\lambda_{x_i}$ is the cascade step size. This relation can be deduced from Equations (6) and (7). Differentiating Equation (7) and using Equation (6) yields

$$\Delta k_z = \frac{G^{1/3}}{3} k_x^{-2/3} \Delta k_x = \frac{k_z}{4n} . \tag{9}$$

Solving the second equality in Equation (9) for Δk_x and using Equation (7) to eliminate k_z from Eq. (19) gives

$$\Delta k_x = \frac{3\, k_x}{4n} . \tag{10}$$

To obtain $\Delta\lambda_x$, one differentiates $k_x = 2\pi/\lambda_x$ obtaining

$$\Delta k_x = \frac{2\pi\,(-1)\,\Delta\lambda_x}{\lambda_x^2} . \tag{11}$$

Hence, from Equations (10) and (11)

$$\Delta k_x = \frac{3k_x}{4n} = \frac{(2\pi)\,3}{\lambda_x\,(4n)} . \tag{12}$$

Therefore, solving Equation (11) for $\Delta\lambda_x$ and using Equation (12)

$$\Delta\lambda_x = \frac{(-3)}{4n}\,\lambda_x . \tag{13}$$

Using Equation (13) in Equation (8) gives

$$\lambda_{x_{i+1}} = \lambda_{x_i} + \Delta\lambda_{x_i} = \lambda_{x_i}\left(1 - \frac{3}{4n}\right) \equiv B\lambda_{x_i} \tag{14}$$

or

$$\lambda_{x_i} = B^i \lambda_{x_o} \tag{15}$$

where i = 0 refers to the scale at the start of the CD (that is to say, the largest break point scale or λ_{max}) and λ_{x_i} is the scale under consideration. The smallest scale would be λ_{min} or λ_J the scale reached at the end of the CD. Thus, in our example λ_{min} or $\lambda_J = 3$ km and λ_{max} or $\lambda_0 = 15$ km. Using Equation (15), then, we obtain

$$B^J = \frac{\lambda_{min}}{\lambda_{max}} . \tag{16}$$

Hence one can obtain the value of J from Equation (16) as

$$J = \frac{\ln\left(\lambda_{min}/\lambda_{max}\right)}{\ln(B)} . \tag{17}$$

One can now evaluate Equation (3) for the CD by using τ_i from Equation (4)

$$CD = \sum_{i=0}^{J} n\tau_i = n\sum_{i=0}^{J} \left(A\,\lambda_{x_i}\right)^{2/} \tag{18}$$

and using Equation (15) for λ_{x_i}

$$CD = n\sum_{i=0}^{J} \left(AB^i\,\lambda_{x_o}\right)^{2/3} = nA^{2/3}\left(\lambda_{x_o}\right)^{2/3} \sum_{i=0}^{J} \left(B^i\right)^{2/3} \tag{19}$$

or

$$CD = nA^{2/3}\left(\lambda_{max}\right)^{2/3} \sum_{i=0}^{J} \left(B^{2/3}\right)^{i}. \tag{20}$$

The sum in Equation (20) is readily evaluated by using the general formula for the sum of a geometric series

$$\sum_{i=0}^{M} a\, r^{i} = a\left(\frac{r^{M}-1}{r-1}\right) \tag{21}$$

(see Burington (1954) p. 3). This gives

$$\sum_{i=0}^{J} \left(B^{2/3}\right)^{i} = \left[\frac{\left(B^{2/3}\right)^{J}-1}{B^{2/3}-1}\right] . \tag{22}$$

Inserting this, the value of A from Equation (5), and the value of B from Equation (14) we finally obtain from Equation (20)

$$CD = n\left(\frac{a_1\, 2\pi}{a_2\, \bar{\varepsilon}}\right)^{1/3} \lambda_{max}^{2/3} \frac{\left[\left[\left(1-\frac{3}{4n}\right)^{2/3}\right]^{J}-1\right]}{\left[\left(1-\frac{3}{4n}\right)^{2/3}-1\right]} . \tag{23}$$

To evaluate this we use λ_{max} = 15 km, λ_{min} = 3 km and, from Table 1, n = 4. This yields B = 0.813 from Equation (14) and J = 7.77 from Equation (17). Thus Equation (23) yields, using the above numbers and the earlier quoted values of $a_2/a_1 = 2.5 \times 10^3$ and $\bar{\varepsilon} = 10^{-5}\,\frac{m^2}{s^2}$, $CD \doteq 22$ hrs . This is to be compared to TOF of Equation (2) since, from theory, CD = TOF . As will be recalled TOF = 24.3 hrs which, in view of the approximate nature of the calculation, is too good an agreement to be expected and must therefore be considered partially fortuitous. If λ_{min} = 2 km had been used, CD = 24.5 hrs.

Figure 1 suggests an additional check on our model. Let us compare TOF and CD for the passage from tropospheric height (10.5 km Nastrom, private communication) to stratospheric height, 13 km. The $\lambda_{max} = 23$ km and $\lambda_{min} = 15$ km. Using the same parametric values as before, $\overline{\varepsilon} = 10^{-5}\ m^2/s^3$, n = 4, $a_2/a_1 = 2.5 \times 10^3$, one finds CD is about 11 hrs. As seen in Table 1 the values of λ_{z_*} are nominally 1 km in the troposphere and 5 km in the stratosphere (we don't have tropopause height information or actual λ_{z_*} information) yielding $v_{g_z} = 1.59 \times 10^{-2}$ m/s and 7.9×10^{-2} m/s respectively. The resulting TOF's are 44 hrs and 9 hrs respectively thus bracketing the CD of 11 hrs. The agreement is therefore within reason.

It should be mentioned that Gao and Meriwether (1998) also measured horizontal wavenumber spectra by aircraft. While Nastrom et al (1985) (1987) measured only horizontal velocity, Bacmeister et al (1996) and Gao and Meriwether (1998) also measured the vertical velocity. In Bacmeister et al (1996) the break in both horizontal velocity and vertical velocity occur at the same scale. This could be explained by saying that this is the scale reached by the cascade. In Gao and Meriwether (1998), in contrast, there is no break at all for the horizontal winds but there is one for the vertical winds. The present model does not explain this, thus one has a new unanswered question requiring further study.

4. FUTURE MEASUREMENTS

To test the model suggested here, the following measurements would be useful. The PSD's should be measured at high and low altitudes at nearby times and places and under similar conditions. The value of $\overline{\varepsilon}$, i.e. the time and volume averaged turbulence dissipation rate at the altitude of the aircraft should be measured. It would also be useful to measure λ_{z_*} (c.f. Smith et al. (1987)). The presence of gravity wave sources such as jet streams, thunderstorms, and wind over mountains should also be documented. In this way one could verify the predicted association between spectral break and GW source. Of course the direct tests of the saturated cascade hypothesis suggested in Dewan (1997) should also be very useful if not of primary value.

5. CONCLUSION

Experimental data published by Nastrom et al (1987) and Bacmeister et al (1996) give some quantitative support to the cascade hypothesis as contained in the S-C model in Dewan (1997). Further tests of this type would entail simultaneous measurements of PSD's and $\bar{\varepsilon}$'s over similar locations and conditions.

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